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TECHNICAL NOTES

FORCED CONVECTIVE HEAT TRANSFER IN THE ENTRANCE REGION OF A PARALLEL PLATE CHANNEL

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NOMENCLATURE

h	channel half depth
Nu_x	local Nusselt number based on channel depth
Re	based on channel depth, $(U_0 2h)/\nu$
T_B	fluid bulk temperature
T_0	fluid temperature at inlet
T_∞	fluid temperature on duct centre-line
T_{ref}	$(T_w - T_0)$ for $T_w = C$, $q_w h/k$ for $Q_w = C$
$\bar{u}$	axial velocity non-dimensionalized by centre-line value

Greek symbols

δ	hydrodynamic boundary layer thickness, $\delta_1 = \delta/h$
δ_t	thermal boundary layer thickness, $\delta_{t_1} = \delta_t/h$
θ_{B_1}	dimensionless bulk temperature, $(T_w - T_\infty)/(T_w - T_B)$
γ	dimensionless centreline temperature, $(T_\infty - T_0)/T_{ref}$

1. INTRODUCTION

LAMINAR convective transport in the entrance region has received considerable attention because in several internal flow situations the length to diameter ratio is not adequate to ensure fully developed conditions. Leading edge effects result in high transport rates. Immediately downstream of the entry, the flow is laminar, irrespective of the diameter based Reynolds number, thereby allowing theoretical evaluations to be made.

The available literature seems to suffer from two shortcomings: the hydrodynamic model, and the centre-line temperature.

Schiller's parabolic velocity distribution, which constrains the flow to be hydrodynamically developed at the axial location of the meeting of the boundary layers, has been generally adopted [1–3]. A parabolic velocity profile is inadequate since it cannot account for the variation of the pressure gradient in the entrance region. A pure viscous filled region is necessary to attenuate the maximum acceleration reached at the end of the boundary layer (inlet) zone [4, 5].

Secondly, the assumption of an invariant centre-line temperature in the development zone [1–3] is unrealistic for low Prandtl number fluids, since the thermal development would be delayed beyond the meeting of the boundary layers (thermal), due to hydrodynamic non-similarities.

The thermal development can occur under three distinct situations: (i) The thermal boundary layers developing faster than the hydrodynamic for low Pr fluids; (ii) In case of moderate Prandtl number, $\delta_t < \delta$; yet the thermal boundary layers meeting within the filled region; (iii) The meeting of δ_ts taking place outside the hydrodynamic entrance region for high Pr fluids.

In the present study the simultaneous development in the entrance of a parallel plate channel is analysed incorporating the two-zone hydrodynamic model [4, 5] and the centre-line temperature variation. Both isothermal wall ($T_w = C$) and uniform heat flux ($Q_w = C$) conditions are considered.

2. ANALYSIS

The pressure gradient controlled velocity profile in the entrance region is given by [5]

$$\bar{u} = (2\eta - 2\eta^3 + \eta^4) + \frac{\lambda}{6}(\eta - 3\eta^2 + 3\eta^3 - \eta^4) + \frac{\Gamma}{2}(\eta^2 - 2\eta^3 + \eta^4) \quad (1)$$

where

$$\eta = y/\delta; \quad \lambda = \frac{\delta^2}{\nu}\frac{dU_\infty}{dx}; \quad \Gamma = \left(\frac{\partial^2 u}{\partial y^2}\right)_{y=h}\frac{h^2}{U_c}$$

are the two pressure gradient parameters. $\Gamma = 0$ in the inlet region.

The choice of a temperature profile was made on the basis of the consideration that the best one would reproduce the fully developed Nusselt number to utmost accuracy. Defining $\theta = (T - T_\infty)/(T_w - T_\infty)$, the profiles are:

$T_w = C$

$$\theta = (1 - \tfrac{3}{2}\eta_1 + \tfrac{1}{2}\eta_1^3) \quad (2)$$

satisfying the boundary conditions

$$\eta_1 = 0: \theta = 1, \quad \frac{\partial^2\theta}{\partial\eta_1^2} = 0,$$

$$\eta_1 = 1: \theta = 0, \quad \frac{\partial\theta}{\partial\eta_1} = 0.$$

$Q_w = C$

$$\theta = 1 - \frac{q_w\delta_t}{k(T_w - T_\infty)}[\eta_1 - \tfrac{1}{2}\eta_1^3 + \tfrac{1}{8}\eta_1^4] \quad (3)$$

constrained by the conditions

$$\eta_1 = 0: \theta = 1, \quad \frac{\partial\theta}{\partial\eta_1} = -\frac{q_w\delta_t}{k(T_w - T_\infty)}, \quad \frac{\partial^2\theta}{\partial\eta_1^2} = 0,$$

$$\eta_1 = 1: \frac{\partial\theta}{\partial\eta_1} = 0, \quad \frac{\partial^3\theta}{\partial\eta_1^3} = 0$$

where

$$\eta_1 = y/\delta_t.$$

The conservation of thermal energy across a control volume coinciding with the interior of the channel walls is expressed in

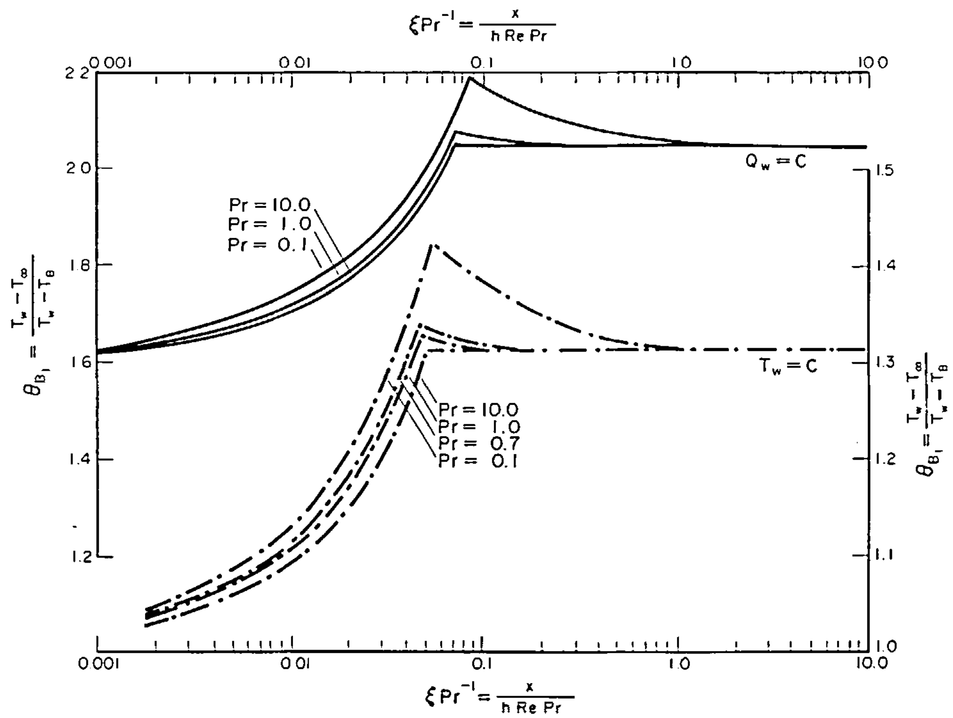

FIG. 1. Variation of dimensionless bulk temperature.

terms of bulk temperature variation as

$$\dot{m}C_p \frac{dT_B}{dx} = q_w P \tag{4}$$

where P is the heat transfer perimeter and q_w the wall heat flux. The general expression for equation (4), upon substitution of u and θ, and integration, results in an ordinary non-linear differential equation of the form

$$A\frac{d\delta_1}{d\xi} + B\frac{d\lambda}{d\xi} + C\frac{d\Gamma}{d\xi} + D\frac{d\delta_{t_1}}{d\xi} + E\frac{d\gamma}{d\xi} = F \tag{5}$$

where ξ $[=x/(h\,Re)]$ is the non-dimensional axial distance. The coefficients A to F are functions of δ, λ, Γ, δ_t and γ. The functions differ for $\delta_t > \delta$ or $\delta_t < \delta$, likewise for $T_w = C$ or $Q_w = C$ condition. The expressions are available in ref. [6].

Using the hydrodynamic results of ref. [5], we evaluate δ_{t_1} in the thermal boundary layer zone and T_∞ in regions thereafter.

At the end of the inlet region $\lambda = 3.10$ and $\Gamma = 0.0$ [5]. The energy equation suggests that the thermal boundary layers will meet at the end of the inlet region if $Pr = 0.6538$ for $T_w = C$ or $Pr = 0.4335$ for $Q_w = C$ condition.

Equation (5) was solved numerically using a 4th order Runge–Kutta method with initial values: $\xi = 0.0001$, $\delta_1 = 0.02$, $\lambda = 0.017$, $\gamma = 0.0$, $\beta = Pr^{-0.33}$, where $\beta = \delta_{t_1}/\delta_1$.

Table 1. Heat transfer results in entry region

	$Pr = 0.1$		$Pr = 1.0$		$Pr = 10.0$	
ξ	Nu_x	$(T_\infty - T_0)/T_{ref}$	Nu_x	$(T_\infty - T_0)/T_{ref}$	Nu_x	$(T_\infty - T_0)/T_{ref}$
(a) Constant wall temperature						
0.002	5.566	0.0	11.603	0.0	26.197	0.0
0.0053	4.280*	0.0	7.913	0.0	16.930	0.0
0.029	4.015	0.645	4.420	0.0	8.495	0.0
0.048	3.985	0.835	3.977*	0.0	7.250	0.0
0.057	3.974†	0.885	3.973†	0.037	6.854	0.0
0.470	3.934	1.000	3.934	0.812	3.976†	0.0
(b) Constant heat flux						
0.002	6.893	0.0	15.330	0.0	31.740	0.0
0.0085	4.379*	0.0	8.378	0.0	17.471	0.0
0.020	4.242	0.296	5.912	0.0	12.058	0.0
0.057	4.156†	0.618	4.352	0.0	8.248	0.0
0.071	4.143	0.674	4.146*	0.0	7.647	0.0
0.660			4.117†	0.653	4.158	0.0

* Indicates meeting of thermal boundary layers.

† Indicates 99% attainment of fully developed θ_{B_1} value.

3. RESULTS AND DISCUSSIONS

The variation of the non-dimensional bulk temperature θ_{B_1} is presented in Fig. 1. It is observed that θ_{B_1} continues to vary beyond the location of $\delta_{t_1} = 1$ for $Pr = 0.1$ and 1.0 corresponding to cases (i) and (ii). The thermal development is delayed till the flow is hydrodynamically developed.

Numerical corroborations are noted in Tables 1(a) and (b). The thermal boundary layers meet at $\xi = 0.0053$ whereas the 99% invariance of θ_{B_1} is attained at $\xi = 0.057$ for $Pr = 0.1$ ($T_w = C$). In case of $Pr = 1.0$, θ_{B_1} attainment is again at $\xi = 0.057$, although the meeting of δ_ts was at $\xi = 0.048$. The corresponding values for the constant heat flux boundary are given in Table 1(b).

Variation of the free stream temperature in the thermal development region are observed for $Pr = 0.1$ and 1.0. Apart from confirming the needed corrections, the γ values are helpful in deciding the useful length of a heating duct. For example, an isothermal duct of $\xi = 0.47$ is not wholly useful for $Pr = 0.1$, whereas the full length is effective for a fluid with $Pr = 10.0$ [Table 1(a)].

As long as the thermal boundary layers meet in the fully developed hydrodynamic zone, as for example when $Pr = 10.0$ (case iii), the boundary layer meeting criterion is synonymous with full thermal development.

In this light the values of thermal entrance lengths as $\xi Pr^{-1} = 0.051$ ($T_w = C$) and 0.069 ($Q_w = C$) reported by Bhatti and Savery [2, 3] stand reviewed for low Prandtl numbers.

The average heat transfer rates for the constant wall temperature computed in the manner

$$\overline{Nu} = \frac{1}{\xi}\int_0^{\xi} Nu_x \, dx,$$

following Kays [7], are summarized through correlations, within accuracies of $\pm 10\%$. These are

$$\overline{Nu} = 3.77 + 0.25\xi^{-0.78} Pr^{0.38} \quad \text{for} \quad 0.1 \leqslant Pr \leqslant 3.0, \quad (6)$$

$$\overline{Nu} = 3.77 + 0.25\xi^{-0.78} Pr^{(3\xi/Pr)^{0.2}}, \quad \text{for} \quad 0.65 \leqslant Pr \leqslant 10.0. \quad (7)$$

The correlation given at (6) valid for all gases and water, is particularly simple and affords convenient design calculations.

4. CONCLUSIONS

The salient features of the present study of simultaneous hydrodynamic and thermal developments in a parallel plate channel are (i) the inclusion of the hydrodynamic filled region, and (ii) accounting for variation of the centre-line temperature after the meeting of the thermal boundary layers. The latter is particularly important for low Pr fluids.

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VAPOUR BUBBLE FORMATION DURING FAST TRANSIENT BOILING ON A WIRE

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1. INTRODUCTION

TRANSIENT boiling phenomena have been receiving considerable attention due to their significance to the safety of water-cooled nuclear reactors. A number of researchers have tackled this problem before, and have used a variety of experimental techniques ranging from heating a thin metallic strip by a laser beam [1], to electrically heated foils [2, 3] and wires [4, 5] immersed in water and various organic fluids. Perhaps the most relevant work to that described here is the contribution by Sakurai [6]. The author used a platinum wire heated electrically by exponentially increasing power, thus simulating a step input of reactivity in a nuclear reactor.

The purpose of this study was to investigate the heat transfer mechanisms accompanying rapid heating of a fine platinum wire (0.025 mm in diameter) immersed in sub-cooled water at a range of pressures up to 14 bar and of rates of increase of the heating surface temperature up to 10^7 K s^{-1} [7]. With such rapid heating, especially at elevated pressures, nucleation occurs in a homogeneous rather than heterogeneous manner. The results presented here, however, were obtained from transient boiling tests with relatively low heating rates, i.e. below 10^6 K s^{-1} and at atmospheric pressure. Such rates of heating were found to lead to stable nucleate boiling following a brief period of local and highly unstable film boiling.

2. EXPERIMENTAL APPARATUS AND PROCEDURE

The apparatus is shown in Fig. 1. Its primary part is a test heater in the form of a platinum wire 0.025 mm in diameter and